1st SEM
(Paper -1)

LESSON PLAN

Class: ~~[illegible]~~ 1st sem Sub: physics No. of Periods / Weeks:

Sl.No.	Month	Paper/ Unit	Topics assigned	Page No
1	2	3	4	5
	December	P-1 Unit II	Vector Algebra Recapitulation of vectors:- properties of vectors under rotations. scalar product and its invariance under rotations, vector product. scalar triple product and their interpretation in terms of area and volume respectively, scalar and vector field. Calculus - II Calculus function more than one variable - Partial derivative Integral factor, constraint of Maximization using lagrange multipliers	

PROGRESS

No.	Date	Time	Topics covered (If class not taken, mantion the reasons)	Signature of Teacher
	2	3	4	5
	15.12	12-1	Introduction of Vector algebra with ideas of Self study	Bkh
	16.12	do	Revision properties of vector	Bkh
	17.12	do	Revision Scalar ~~product~~ product	Bkh
	18.12	do	Vector triple product	Bkh
	19-12	-do-	Variance under rotation	Bkh
			21/12/2020	
	20.12	do	Briefly discussion and Revision about calculus-II	Bkh
	21.12	do	Doubt clear with some question	Bkh

N.B.: At the end of every week, It is be countersigned by HOD and the end of every month; It is to countersigned by the Principal.

Funⁿ of more than one Variable :-

Limits :- If $z = f(x,y)$ be a funⁿ of two variable then it is said to have limit L as $x \to a$, $y \to b$

$$\boxed{\therefore \lim_{(x,y)\to(a,b)} f(x,y) = L}$$

$\lim_{\substack{x\to a\\ y\to b}} f(x,y) = L$, $\lim_{\substack{x\to a\\ y\to b}} g(x,y) = m$

(1) $\lim_{\substack{x\to a\\ y\to b}} (f \pm g) = L \pm m$ (2) $\lim_{\substack{x\to a\\ y\to b}} (f\cdot g) = Lm$

(3) $\lim_{\substack{x\to a\\ y\to b}} \left(\frac{f}{g}\right) = \frac{L}{m}$ if $m \neq 0$

Continuity :- The funⁿ $f(x,y)$ is said to be continuous at point (a,b) if $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$.

Working of Limit :-

St-1 find $f(x,y)$ along $x \to a$ & $y \to b$

St-2 find $f(x,y)$ along $y \to b$ & $x \to a$

St-3 If $a \to 0$, $b \to 0$ find limit along $y = mx$ or $y = mx^2$.

Ex-1 Find Limit $\lim_{(x,y)\to(0,0)} \frac{xy}{y^2 - x^2}$

Solⁿ $\lim_{\substack{x\to 0\\ y\to 0}} \frac{xy}{y^2 - x^2} = \lim_{y\to 0} 0 = 0$

$\lim_{x\to 0} = 0$

$y = mx$ along the line

$\lim_{x\to 0} \frac{mx^2}{m^2x^2 - x^2} = \lim_{x\to 0} \frac{x^2 m}{x^2(m^2-1)} = \frac{m}{m^2-1}$ ∴ Limit doesn't exist

∴ it differ from value of m

Ex-2 find the value of $\lim_{\substack{x\to 1\\ y\to 2}} \frac{2x^2y}{x^2+y^2+1}$

$\lim_{\substack{x\to 1\\ y\to 2}} \frac{2x^2y}{x^2+y^2+1} = \lim_{x\to 1} \frac{2\cdot 2x^2}{x^2+4+1} = \frac{4x^2}{x^2+5} = \frac{4}{6} = \frac{2}{3}$

Ex-4 Show that the fun^n $f(x,y) = x^2+2y \quad (x,y) \neq (1,2)$
$\qquad\qquad\qquad\qquad = 0 \quad , (x,y) = (1,2)$ is continuous at $(1,2)$

$\lim\limits_{\substack{x\to 1\\ y\to 2}} f(x,y) = \lim\limits_{\substack{x\to 1\\ y\to 2}} (x^2+2y) = \lim\limits_{y\to 2} (1+2y) = 5$

$\lim\limits_{\substack{x\to 1\\ y\to 2}} (x^2+2y) = \lim\limits_{x\to 1} (x^2+4) = 5$ But $f(x,y) = 0$ $x=1, y=2$.

PARTIAL DERIVATIVES:-

If $z = f(x,y)$ then derivative of z w.r.t x keeping const is called partial derivative of z w.r.t x

$$\frac{dz}{dx} = \lim_{\delta x\to 0} \frac{f(x+\delta x, y) - f(x,y)}{\delta x} = f_x(x,y)$$

Also derivative of z w.r.t y keepin x const.

$$\frac{dz}{dy} = \frac{f(x, y+\delta y) - f(x,y)}{\delta y} = f_y(x,y).$$

PARTIAL DERIVATIVES OF HIGHER ORDER:-

If $z = f(x,y)$ then $\frac{d}{dx}\left(\frac{dz}{dy}\right) = \frac{d^2z}{dxdy}$ or f_{xy}

$\qquad\qquad\qquad\qquad\quad \frac{d}{dy}\left(\frac{dz}{dy}\right) = \frac{d^2z}{dy^2}$ or f_{yy}

Ex 1. find first order derivatives of $u = e^x \sin y$

sol^n:- $u = e^x \sin y$

$\Rightarrow u_x = \frac{d(e^x \sin y)}{dx} = e^x \sin y \qquad u_y = \frac{d\, e^x \sin y}{dx} = e^x \cos y.$

Ex 2

if $z = \ln(x+\sqrt{x^2-y^2})$ find first order derivatives.

$z = \ln(x+\sqrt{x^2-y^2})$

$$\therefore \frac{dz}{dx} = \frac{1}{x+\sqrt{x^2-y^2}}\left(1+\frac{1}{\cancel{2}\sqrt{x^2-y^2}}(\cancel{2}x)\right)$$

$$= \frac{1}{x+\sqrt{x^2-y^2}}\left(1+\frac{x}{\sqrt{x^2-y^2}}\right)$$

$$\frac{dz}{dy} = \frac{1}{x+\sqrt{x^2-y^2}}\left(\frac{1}{\cancel{2}\sqrt{x^2-y^2}}(-\cancel{2}y)\right)$$

$$= \frac{-y/\sqrt{x^2-y^2}}{x+\sqrt{x^2-y^2}}$$

variable to be Treated as constant :-

If $z = x^2 + 2y^2$ and $x = r\cos\theta$, $y = r\sin\theta$. find $\left(\frac{dz}{dy}\right)_r$, $\left(\frac{dz}{d\theta}\right)_x$

we have $x^2 + y^2 = r^2$

$\tan\theta = \frac{y}{x} \Rightarrow y = x\tan\theta$

$\therefore z = x^2 + 2y^2$

putting $x^2 = r^2 - y^2$

we get $z = r^2 - y^2 + 2y^2$

$\Rightarrow z = r^2 + y^2 \Rightarrow \left(\frac{dz}{dy}\right) = 2y$

Also $z = x^2 + 2y^2$

$= x^2 + 2x^2 \tan^2\theta$

$\left(\frac{dz}{d\theta}\right) = 2x^2 (2\tan\theta)\sec^2\theta = 4x^2 \tan\theta \frac{r^2}{x^2}$ ($\sec\theta = \frac{r}{x}$)

$= 4r^2 \tan\theta$

Total Derivative :-

if $z = f(x, y)$ be funⁿ of 2 variables then total diff is represent as $dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$

Similarly $f = f(x, y, z)$ then $df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$

$$\frac{du}{dt} = \frac{du}{dx} \cdot \frac{dx}{dt} + \frac{du}{dy} \cdot \frac{dy}{dt}.$$

Q1 if $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ PT $x^2 \frac{du}{dx} + y^2 \frac{du}{dy} + z^2 \frac{du}{dz} = 0.$

let $s = \frac{y-x}{xy} = \frac{1}{x} - \frac{1}{y}$ so $\frac{ds}{dx} = \frac{-1}{x^2}$, $\frac{ds}{dy} = \frac{1}{y^2}$, $\frac{ds}{dz} = 0$

$t = \frac{z-x}{xz} = \frac{1}{x} - \frac{1}{z}$; $\frac{dt}{dx} = -\frac{1}{x^2}$, $\frac{dt}{dy} = 0$, $\frac{dt}{dz} = \frac{1}{z^2}$

we have

$u = u(s, t)$

$\Rightarrow \frac{du}{dx} = \frac{du}{ds} \cdot \frac{ds}{dx} + \frac{du}{dt} \cdot \frac{dt}{dx} = \frac{du}{ds}\left(-\frac{1}{x^2}\right) + \frac{du}{dt}\left(-\frac{1}{x^2}\right)$

$\Rightarrow x^2 \frac{du}{dx} = -\frac{du}{ds} - \frac{du}{dt}$ ——— (1)

Similarly $\frac{du}{dy} = \frac{du}{ds} \cdot \frac{ds}{dy} + \frac{du}{dt} \cdot \frac{dt}{dy} = \frac{du}{ds}\left(\frac{1}{y^2}\right) + \frac{du}{dt} \cdot 0$

$y^2 \frac{du}{dy} = \frac{du}{ds}$ ——— (2).

Now $\frac{du}{dz} = \frac{du}{ds}\cdot\frac{ds}{dz} + \frac{du}{dt}\cdot\frac{dt}{dz} = \frac{du}{ds}(0) + \frac{du}{dt}\left(\frac{1}{z^2}\right)$

$= z^2\frac{du}{dz} = \frac{du}{dt}$ — (3)

adding eq^n 1, 2, & 3

$$x^2\frac{du}{dx} + y^2\frac{du}{dy} + z^2\frac{du}{dz} = -\frac{du}{ds} - \frac{du}{dt} + \frac{du}{ds} + \frac{du}{dt}.$$

$$x^2\frac{du}{dx} + y^2\frac{du}{dy} + z^2\frac{du}{dz} = 0.$$

Euler's Theorem:-

If u is a homogeneous fun^n of degree n in x and y then Euler's theorem is $x\frac{du}{dx} + y\frac{du}{dy} = nu$.

Ex-1

$u = \sin^{-1}\frac{x^2+y^2}{x+y}$ PT $x\frac{du}{dx} + y\frac{du}{dy} = \tan u$.

let $f = \sin u = \frac{x^2+y^2}{x+y} = \frac{x^2(1+y^2/x^2)}{x(1+y/x} = x\phi(y/x)$ $n=1$

from Euler's theorem $x\frac{df}{dx} + y\frac{df}{dy} = nf$.

$$\Rightarrow x\cos u\frac{du}{dx} + y\cos u\frac{du}{dy} = f$$

$$\Rightarrow \cos u\left[x\frac{du}{dx} + y\frac{du}{dy}\right] = \sin u.$$

$$\Rightarrow x\frac{du}{dx} + y\frac{du}{dy} = \frac{\sin u}{\cos u} = \tan u.$$

Ex 2

$u = \log(x^2+xy+y^2)$ PT $x\frac{du}{dx} + y\frac{du}{dy} = 2.$

sol^n let $f = e^u = x^2+xy+y^2 = x^2\left(1+\frac{xy}{x^2}+\frac{y^2}{x^2}\right)$

$= x^2\left(1+\frac{y}{x}+\frac{y^2}{x^2}\right) = x^2\phi\left(\frac{y}{x}\right.$

degree $= 2$

Applying Euler's thm $= x\frac{df}{dx} + y\frac{df}{dy} = nf$

$$\Rightarrow xe^u\frac{du}{dx} + ye^u\frac{du}{dy} = 2e^u \{f = e^u$$

$$\Rightarrow x\frac{du}{dx} + y\frac{du}{dy} = 2.$$

Exact Diff. Eq^n:-

if the eq^n is in form of $M(x,y)\,dx + N(x,y)\,dy = 0$ is said to be exact diff eq^n.

condition $\boxed{\frac{dM}{dy} = \frac{dN}{dx}}$

Ex 1 $\quad y\,dx + x\,dy = 0$

$M\,dx + N\,dy = 0$

$M = y \quad N = x$

$\frac{dM}{dy} = 1 \quad \frac{dN}{dx} = 1$

$\therefore \frac{dM}{dy} = \frac{dN}{dx} \Rightarrow$ The eq^n is exact.

Integrating factor:-

1) $x\,dy + y\,dx = d(xy)$

2) $\frac{y\,dx - x\,dy}{y^2} = d\left(\frac{x}{y}\right)$

3) $\frac{x\,dy - y\,dx}{x^2} = d\left(\frac{y}{x}\right)$

4) $\frac{y\,dx - x\,dy}{x^2+y^2} = d\left(\tan^{-1}\frac{x}{y}\right)$

5) $\frac{y\,dx - x\,dy}{xy} = d\left(\log\frac{x}{y}\right)$

(6) $\frac{y\,dx - x\,dy}{x^2y^2} = d\left(\frac{1}{xy}\right)$

(7) $\frac{ye^x\,dx - e^x\,dy}{y^2} = d\left(\frac{e^x}{y}\right)$

(8) $\frac{2x\,dx + 2y\,dy}{x^2+y^2} = d\left[\log(x^2+y^2)\right]$

(9) $\frac{2xy\,dx - x^2\,dy}{y^2} = d\left(\frac{x^2}{y}\right)$

(10) $\frac{2xy^2\,dx - 2yx^2\,dy}{y^4} = d\left(\frac{x^2}{y^2}\right)$.

Integrating factor of Non Exact Diff Eq^n:-

Rule 1 $\quad$ if $Mx + Ny \neq 0$ then $\frac{1}{Mx+Ny}$ is IF.

$M\,dx + N\,dy = 0$

Rule 2 $\quad$ if $M = yf_1(xy)$, $N = xf_2(xy)$

$DF = f_1(xy)\,y\,dx + f_2(xy)\,x\,dx = 0$

then $\frac{1}{Mx - Ny}$ is IF.

Rule 3 $\quad$ if $\left(\frac{dM}{dy} - \frac{dN}{dx}\right)/N$ in a fun^n of x only $= F(x)$ then IF

Rule 4 $\quad \left(\frac{dN}{dx} - \frac{dM}{dy}\right)/M$ is fun^n of y: $f(y)$ is IF;

$IF = e^{\int f(y)\,dy}$.

Rule-V :- if . DE is in the form of $x^a y^b (mydx + nxdy)$ $+ x^{a'} y^{b'} (m'ydx + n'xdy) = 0$ the IF $= x^h y^k$.

LAGRANGE'S METHOD :-

1- if $F(x,y,z) = f(x,y,z) + \lambda \phi(x,y,z)$

2- Parti diff $F(x,y,z)$ wrt x,y,z equat them to zer

$\frac{dF}{dx} = 0, \frac{dF}{dy} = 0 \quad \frac{dF}{dz} = 0.$

3- solve four eq^n $\frac{dF}{dx} = 0 \; \frac{dF}{dy} = 0 \; \frac{dF}{dz}$ and $\phi(x,y,z) = 0$.

to obtained value of $x \; y \; \& \; z$.

Ex-1

find minimum value of $x^2+y^2+z^2$ condition $xyz = a^3$

$f(x,y,z) = x^2+y^2+z^2$.

contraint eq^n is $\phi(x,y,z) = xyz - a^3$

$\therefore$ A.E $F(x,y,z) = f(x,y,z) + \lambda \phi(x,y,z)$

$F(x,y,z) = x^2+y^2+z^2 + \lambda(xyz - a^3)$ —— (1)

Then $\frac{dF}{dx} = 0$ gives $2x + \lambda yz = 0$ — (2)

$\frac{dF}{dy} = 0$ gives $2y + \lambda xz = 0$ —— (3)

$\frac{dF}{dz} = 0$ gives $2z + \lambda xy = 0$ —— 4.

from eq^n (2) (3) & (4) we get $2x = -\lambda yz$ or $2x^2 = -\lambda xyz$

$2y = -\lambda xz$, $2z = -\lambda xy$

$\therefore$ putting the value of x, y, z in $xyz = a^3$

$x = a \quad y = a \quad z = a$.

Hence min^m value of funct^n is $x^2+y^2+z^2 = 3a^2$.

Vector Algebra

vector < both — magnitude, dir^n

scalar — mag

$\vec{A}$ $|\vec{A}|$. magnitud

- equal vector :

$\vec{A}$ & $\vec{B}$

same dir^n & magn. $\longrightarrow \vec{A}$

$\longrightarrow \vec{B}$.

$|\vec{A}| = |\vec{B}|$

- Negative vector

equal magnitud but opposite dir^n

$|\vec{A}| = |-\vec{B}|$

$\vec{A}$ $\longrightarrow$

$\longleftarrow$ $\vec{B}$

$\vec{A} = -\vec{B}$.

- Null vector

if magnitude is zero and dir^n - arbitrary

- co-liner vectors

parallel vector & Anti par — same — dir^n oppo — magnitude may be equal or not

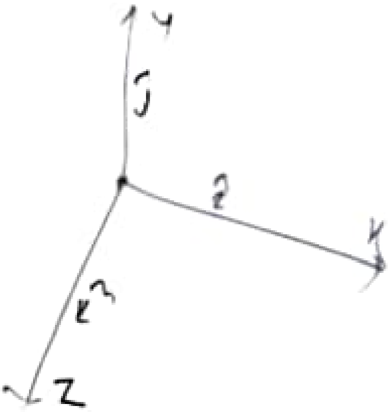
Unit vector

$\vec{A}$

$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$ A vect m.

$\hat{A} = \frac{\vec{A}}{A}$

$\hat{A}$



Like Vectors

Polar Vector

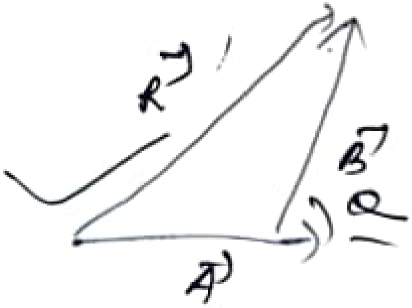
Addition of two Vectors.

Let $\vec{A}, \vec{B}$.

Result

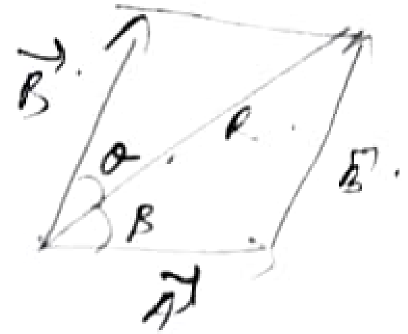
$\vec{R} = \vec{A} + \vec{B}$

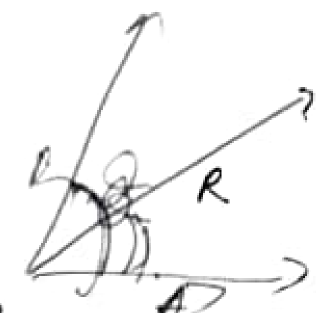
Triangle law of Vector Addn.



[illegible] Law of Vector Addition

90%





$R = \sqrt{A^2+B^2+2AB\cos\theta}$

$\tan\beta = \dfrac{B\sin\theta}{A+B\cos\theta}$

$\beta = \tan^{-1}\left(\dfrac{B\sin\theta}{A+B\cos\theta}\right)$

θ $\vec{A}$ & $\vec{B}$

β $\vec{A}$ & $\vec{R}$

$\dfrac{1}{\tan} = \tan^{-1}$

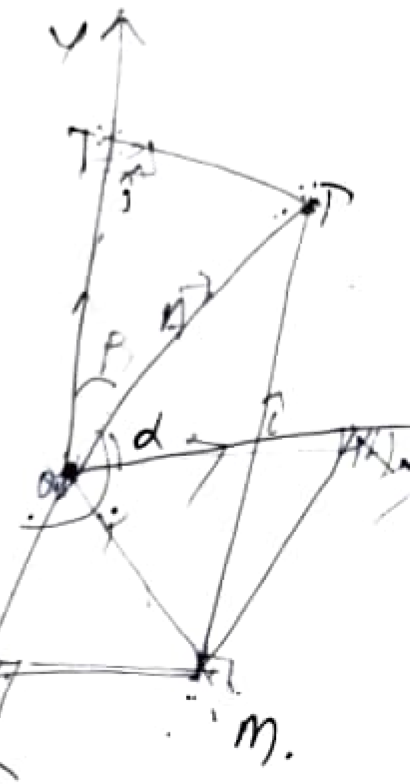
$\dfrac{1}{a^n} = a^{-n}$

$\vec{A}+\vec{B} = \vec{B}+\vec{A}$

$\vec{A}+(\vec{B}+\vec{C}) = (\vec{A}+\vec{B})+\vec{C}$

$= \vec{A}+\vec{B}+\vec{C}$

commutative law & associative



$\vec{OP} = \vec{r}$

$\vec{ON} = \vec{A_x}$

$OT = \vec{A_y}$

$OS = \vec{A_z}$

$\vec{OP} = \vec{OM} + \vec{OT}$

or $OM + \vec{OT} = \vec{OP}$ —(1)

$OS + SM = OM$

$OS + ON = OM$ —(2)

$\vec{OP} = \vec{OS} + \vec{ON} + \vec{OT}$

$= \vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$

$r = \sqrt{x^2 + y^2 + z^2}$

$\hat{n} = \dfrac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

$(2i - 3j - 2k)$

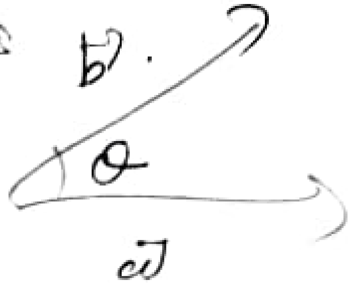
$\vec{A}\cdot\vec{B} = AB\cos\alpha$ Scalar product

$i\cdot j = 0$, $j\cdot k = 0$, $k\cdot i = 0$

$i\cdot i = 1$, $j\cdot j = 1$, $k\cdot k = 1$

$\vec{a}$ & $\vec{b}$ be are two non-zero vectors
& θ be the ap

scalar product of two vectors



$\vec{a}\cdot\vec{b}$ is dn.

$|\vec{a}|\,|\vec{b}|\cos\theta$.

$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\alpha$ dot pr.

$\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{b} = 8\hat{i} + \hat{j}$.

scalar compe b on $\vec{a}$

$\cos\theta \cdot \hat{a}$ $= |\vec{b}| \cdot \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$.

$10/2$

$\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{a}|}$

$\frac{8+2-0}{\sqrt{9}} = \frac{10}{3}$

vectorm $\vec{b}$ on $\hat{a}$

$|\vec{b}|\cos\theta\,\hat{a}$

$\sqrt{65}$

$\vec{a}$ on $\vec{b}$ where

$\vec{a} = \hat{i} - \hat{j} - \vec{k}$

$\vec{b} = 3\hat{i} + \hat{j} + 3\vec{k}$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{3-1-3}{\sqrt{3}\sqrt{19}} = -\frac{1}{\sqrt{3}\sqrt{19}}$$

scalar projection $\vec{a}$ on $\vec{b}$

$$= |\vec{a}|\cos\theta = \sqrt{3}\,\frac{-1}{\sqrt{3}\sqrt{19}} = \frac{-1}{\sqrt{19}}$$

vect pn $\vec{a}$ on $\vec{b}$

$$= \underline{|\vec{a}|\cos\theta}\,.\,\vec{\hat{b}}$$

$$= \frac{-1}{\sqrt{19}}\cdot\frac{\vec{b}}{|\vec{b}|} = \frac{-1}{\sqrt{19}}\;\frac{3\hat{i}+\hat{j}+3\vec{k}}{\sqrt{19}}$$

$$\vec{a}\cdot\vec{b} = 0 \qquad = \frac{-3\hat{i}-\hat{j}-3\vec{k}}{19}$$

$\vec{AB} = -2\hat{i} - 3\hat{j} - 4\hat{k}$

$\vec{CD} = 3\hat{i} + 6\hat{j} - 6\hat{k}$

$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a}$

$|\vec{a}| = |\vec{b}| = |\vec{c}|$

$\vec{AB}\cdot\vec{CD} = 6$

$-6 - 18 + 24 = 0$

are

$\vec{AB} \perp$ to $\vec{CD}$

$\vec{r}$ = Angsfration Vecton (accu...) alpha sendrome.

$$\vec{a}+\vec{b}+\vec{c}=0$$

$$\vec{a} = -(\vec{b}+\vec{c})$$

$$\vec{a}\cdot\vec{a} = -(\vec{b}+\vec{c})\cdot\vec{a}$$

$$|\vec{a}|^2 = -\vec{b}\cdot\vec{a} - \vec{c}\cdot a.$$

$$|\vec{OA}| = |\vec{OP}| = |\vec{OB}|$$ W.F.S

$$|\vec{a}| = |\vec{r}| = |-\vec{a}|$$

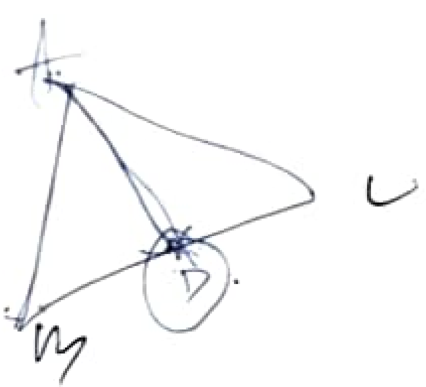
$$\vec{AP} = \vec{OP} - \vec{OA} = \vec{r} - \vec{a}.$$

$$\vec{BP} = \vec{r} + \vec{a}.$$

$$\vec{AP}\cdot\vec{BP} = (\vec{r}-\vec{a})\cdot(\vec{r}+\vec{a})$$

$$= \vec{r}\cdot\vec{r} + \cancel{\vec{r}\cdot\vec{a}} - \cancel{\vec{a}\cdot\vec{r}} + \vec{a}\cdot\vec{a}$$

$$|\vec{r}|^2 - |\vec{a}|^2 = 0$$



3 . 5 . 2003

3 . 4 . 2003

...tor products, (cross prod)

$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$

$i\times i = 0 \quad j\times j = 0$

$i\times j = k \quad j\times k = i$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$

$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}| \sin\theta\, \hat{n}$

$$\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$

$$\sin\theta = \left|\frac{\vec{a}\times\vec{b}}{\vec{a}\times\vec{b}}\right|$$

$$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}|\times|\vec{b}|} \qquad |\vec{c}| = \sqrt{3}$$

$\vec{a} = i + j + k$

$\vec{b} = 4i + 3j + 4k$

$\vec{c} = i + \lambda j + \beta k$

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \lambda & \beta \end{vmatrix}$$

$\vec{AB} = \vec{b} - \vec{a}$

$\vec{BC} = \vec{c} - \vec{b}$

$\vec{CA} = \vec{a} - \vec{c}$

$((\vec{a} \times \vec{b}) + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) \times \vec{AB} = 0$

$(\vec{a} - \vec{d}) \times (\vec{b} - \vec{c})$

$= \vec{a} \times \vec{b} - \vec{a} \times \vec{c} \cancel{\times} - \vec{d} \times \vec{b} + \vec{d} \times \vec{c}$

$= 0$

$\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$

$\vec{a} = 3i + 2j - \hat{k}$

$\vec{b} = -2i + j + k$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 3 & 2 & -1 \\ -2 & 1 & 1 \end{vmatrix}$$

$3 - -4$

$= 3i - j + 7\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{59}$

$(\vec{a} - \vec{b}) [\vec{b} - \vec{c} \times (\vec{c} - \vec{a})]$

$(\vec{a} - \vec{b}) \cdot [\vec{b} \times \vec{c} - \vec{b} \times \vec{a} - \vec{c} \times \vec{c} + \vec{c} \times \vec{a}]$

$(\vec{a} - \vec{b}) [\vec{b} \times \vec{c} - \vec{b} \times \vec{a} + \vec{c} \times \vec{a}]$

$\vec{a} \cdot (\vec{b} \times \vec{c}) - \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (\vec{c} \times \vec{a})$

$- \vec{b} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{b} \times \vec{a})$

$- \vec{b} \cdot (\vec{c} \times \vec{a})]$

$\vec{a} \cdot (\vec{b} \times \vec{c}) - \vec{b} (\vec{c} \times \vec{a}) = 0$

$\vec{AB} = \vec{b} - \vec{a}$
$\vec{BC} = \vec{c} - \vec{b}$
$\vec{CA} = \vec{a} - \vec{c}$

$((\vec{a}\times\vec{b}) + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}) \times \vec{AB} = 0$

$(\vec{a}-\vec{d}) \times (\vec{b}-\vec{c})$

$= \vec{a}\times\vec{b} - \vec{a}\times\vec{c} \;\cancel{\times d} - \vec{d}\times\vec{b} + \vec{d}\times\vec{c}$

$= 0 \qquad \vec{a}\times\vec{b} = \vec{c}\times\vec{d}$

$\vec{a} = 3i + 2j - \hat{k}$

$\vec{b} = -2i + j + k$

$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 3 & 2 & -1 \\ -2 & 1 & 1 \end{vmatrix}$

$3 - -4$

$= 3i - j + 7\hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{59}$

$(\vec{a}-\vec{b})\,[\vec{b}-\vec{c} \times (\vec{c}-\vec{a})]$

$(\vec{a}-\vec{b}) \cdot [\vec{b}\times\vec{c} - \vec{b}\times\vec{a} - \vec{c}\times\vec{c} + \vec{c}\times\vec{a}]$

$(\vec{a}-\vec{b})[\vec{b}\times\vec{c} - \vec{b}\times\vec{a} + \vec{c}\times\vec{a}]$

$\vec{a}\cdot(\vec{b}\times\vec{c}) - \vec{a}\cdot(\vec{b}\times\vec{a}) + \vec{a}\cdot(\vec{c}\times\vec{a})$

$-\vec{b}\cdot(\vec{b}\times\vec{c}) + \vec{b}\cdot(\vec{b}\times\vec{a})$

$-\vec{b}\cdot(\vec{c}\times\vec{a})]$

$\vec{a}\cdot(\vec{b}\times\vec{c}) - \vec{b}\,(\vec{c}\times\vec{a}) = 0$

~~scalar triple~~ $\rightarrow$ (4-7)

$\vec{a}, \vec{b}$ & $\vec{c}$

$\vec{a}\cdot(\vec{b}\times\vec{c})$ or $(\vec{a}\times\vec{b})\cdot\vec{c}$

$$\boxed{\begin{aligned}\vec{a} &= a_1 i + a_2 \hat{j} + a_3 k \\ \vec{b} &= b_1 i + b_2 j + b_3 k \\ \vec{c} &= c_1 i + c_2 j + c_3 k\end{aligned}}$$

$\vec{a}\cdot(\vec{b}\times\vec{c})$

$(\vec{a}\times\vec{b})\cdot\vec{c}$.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$2y\lambda = -400\,xz^2 \Rightarrow (2y)(-800)\,xyz^2 = -400\,xz^2 \Rightarrow y = \frac{1}{2}$$

Similarly, $z^2 = \frac{1}{2}$

$\therefore$ The highest temperature on the surface of unit sphere.

$$T = 400\,xyz^2 = 400.\frac{1}{2}.\frac{1}{2}.\frac{1}{2} = 50.$$

EXERCISE

1. Find the minimum value of $x^2 + y^2 + z^2$ given that, $ax + by + cz = P$.

   Ans. $\frac{p^2}{a^2} + b^2 + c^2$

2. If $xyz = 8$, find the values of x, y, z for which $u = \frac{5xyz}{x + 2y + 4z}$ is a maximum

   Ans. 4, 2, 1.

3. Find the maximum and minimum distances from the origin to the curve $5y^2 - 8 = 0$.

   Ans. 4, 1

4. Find the dimension of rectangular box of maximum capacity whose surface when (a) box is open (b) box is closed.

   Ans. (a) length = breadth = 2 × height, (b) length = breadth = height.

5. Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

   Ans. $\frac{8abc}{3\sqrt{3}}$

6. Find the maximum value of xyz under the constraint $x^2+z^2=1$ and $y-x=0$.

   Ans. $\frac{2}{3\sqrt{3}}$

7. Find the extreme value of $a^2x^2+b^2y^2+c^2z^2$ such that $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1, a>0, b>0, c>0$.

   Ans. $(a+b+c)^3$

8. Find the critical (stationary values) of function $f(x, y, z)=x^2+y^2+z^2$ given that $z^2=xy+1$.
   Ans. $(0, 0, -1), (0, 0, 1)$

9. Find the shortest and longest distances from the point $(1, 2, -1)$ to $x^2+y^2+z^2=24$.
   Ans. A particle is in a rectangular parallelopiped 60x with sides a, b and c.

10. Find the shape of box which will minimise energy E given by $E=\frac{h^2}{8m}\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)$ if volume is constant.
    Ans. Cube $a=b=c$.

SHORT QUESTIONS

TYPE - I

Fill in the Blanks (*Each Question is of 1 Mark*)

1. If $f(x, y) = e^x \sin y$ then $\frac{\partial f}{\partial x}$ = ________

2. If $u = e^x \cos y$ then $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ = ________

3. If $x + y + z = \log z$, then $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$ = ________

4. If $z = f(ax + by)$, then $b\frac{\partial z}{\partial x} - a\frac{\partial z}{\partial y}$ = ________

5. If $z = \sqrt{x^2 + y^2}$, then value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ = ________

6. If $x = r\cos\theta$, $y = r\sin\theta$, then $\frac{\partial r}{\partial x}$ = ________

7. If $z = x^2 - y^2$, $x = r\cos\theta$, $y = r\sin\theta$ then $\left(\frac{\partial z}{\partial r}\right)_y$ = ________, while $\left(\frac{\partial z}{\partial r}\right)_x$ = ________

8. If u is a homogeneous function of degree n in x and y then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ = ________

9. If $W = f(r, s, t)$, such that $r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x}$ then $x\frac{\partial W}{\partial x} + y\frac{\partial W}{\partial y} + z\frac{\partial W}{\partial z}$ = ________

10. The differential equation $M(x, y)\,dx + N(x, y)\,dy = 0$ is an exact differential equation for which the necessary condition is ________ = ________.

11. If the first order differential equation is not exact then it can be made exact by multiplying it with a quantity known as ________

12. One of the integrating factors of the equation $ydx + xdy = 0$ is ________

13. The equation $\sin x \cos y\, dx + \cos x \sin y\, dy = 0$ is ________ differential equation with solution y ________

TYPE - II

Short Question (*Each question is of 1.5 Mark*)

If $u = (x^2 + y^2 + z^2)^{1/2}$. Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$

If $w = x^2 + y^2$, $x = e^t \cos t$, $y = e^t \sin t$, Find $\frac{dw}{dt}$

Find $\frac{dy}{dx}$ if $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

If $x = r\cos\theta$, $y = r\sin\theta$, then show that $\frac{\partial^2 \theta}{\partial x \partial y} = \frac{-\cos 2\theta}{r^2}$

Solve $xdx + ydy + 2(x^2 + y^2)\,dx = 0$.

Solve $(2x - y)\,dy - (x + 2y)\,dx = 0$.

Find the minimum value of $x^2 + y^2 + z^2$ if $3x + 4y + 5z = 5$.

Find the angle between $\left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$ and $\left(\hat{i} + \hat{j} - \hat{k}\right)$.

Find the area of a parallelogram if its two sides are represented by $\left(3\hat{i} + 2\hat{j} - 2\hat{k}\right)$ and $\left(2\hat{i} - 3\hat{j} + \hat{k}\right)$.

Show that $\left[\vec{A} + \vec{B}, \vec{B} + \vec{C}, \vec{C} + \vec{A}\right] = 2\left[\vec{A}\vec{B}\vec{C}\right]$.

Find out a unit vector which is perpendicular to $2\hat{i} + 3\hat{j} + 4\hat{k}$ and lies in XY plane.

Check if the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$, $-2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$ are coplanar.

Show that $(A.B)^2 + (A \times B)^2 = (AB)^2$

14. Check if the points $(5, 2, -4)$, $(1, 1, 2)$ and $(-3, 0, 8)$ form a triangle.

15. What are vector and scalar fields?

TYPE - III

Short Question (*Each question is of 2.5 Mark*)

1. If $u = [illegible]$ then show that $\frac{\partial^3 u}{[illegible]} = e^{xyz}\left[1 + 3xyz + x^2y^2z^2\right]$